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Technical Note

Natural convection heat and mass transfer in partially heated vertical parallel plates

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Abstract

A combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical parallel plates with unheated entry and unheated exit is presented. Both boundary conditions of uniform wall temperature/uniform wall concentration (UWT/UWC) and uniform heat flux/uniform mass flux (UHF/UMF) are considered. Results of dimensionless induced volume rate Q, average Nusselt number Nu_E and Sherwood number Sh_E are obtained for air flow under various buoyancy ratio N, Grashof number Gr_L, Schmidt number Sc and combinations of unheated entry, heated section and unheated exit length. Theoretical solutions for Q, Nu_E and Sh_E for both UWT/UWC and UHF/UMF cases are derived under fully developed conditions. The numerical solutions are shown to approach asymptotically the closed form solutions for fully-developed flow. The presence of unheated entry and unheated exit severely affects the heat and mass transfer rates. In addition, the correlation equations of Q, Nu_E and Sh_E for both boundary conditions are presented.

1. Introduction

The phenomenon of natural convection heat and mass transfer has received considerable attention due to its many applications in diverse fields such as chemical engineering, environmental dynamics, and architectural design. Most of the previous theoretical and experimental studies are concerned with the boundary conditions of uniform wall temperature and concentration. Gebhart and Pera [1,2] obtained the similarity solutions for air and water over a wide range of the Schmidt number Sc. Schenk et al. [3] repeated some of the calculations of Ref. [1]. They found that heat and mass transfer can be approximated as mutually independent for $0.6 < Sc < 0.9$. A similarity form and numerical results for combined effects of thermal and species diffusion along an inclined flat plate or vertical cylinder were proposed by Chen and Yeh [4,5]. The transient free convection heat and mass transfer along a semi-infinite vertical isothermal plate was studied by Gallahan and Marner [6]. Soundalgekar and Warve [7] proposed an analytical study on the unsteady free convection flow passed an infinite porous plate. Soundalgekar and Ganesan [8] carried out an implicit finite-difference analysis for the same problem studied by Gallahan and Marner [6]. Khair and Bejan [9] obtained the similarity solution for a wide spectrum of fluid properties and developed accurate Sherwood number predictions for both large and small values of Pr, Sc and Le. Mollendorf and Gebhart [10] presented a similarity solution for double-diffusive free convection between a parallel plate with asymmetric mass transfer.

A change of boundary conditions from uniform wall temperature to uniform heat flux leads to a quite

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Nomenclature

a	constant in velocity profile equation
b	half channel width [m]
C, c	dimensionless and dimensional species mass fraction, $C=(c-c_0)/(c_w-c_0)$ for UWT/UWC or $(c-c_0)/(m_w b/D)$ for UHF/UMF
D	mass diffusivity [m^2 s^{-1}]
E	ratio of discrete section length to channel length
g	gravitational acceleration [m s^{-2}]
Gr_E	Grashof number based on the heated section length, Gr_L/E_2
Gr_L	Grashof number based on the full channel length, $g\beta\Delta Tb^4/(l\nu^2)$
k	thermal conductivity [W m^{-1} K^{-1}]
L, l	dimensionless and dimensional length of the vertical duct, $L = 1/Gr_L$
Le	Lewis number, Sc/Pr
m	mass flux [kg s^{-1} m^{-2}]
N	buoyancy ratio, $\beta^*(c_w-c_0)/\beta(T_w-T_0)$ for UWT/UWC or $(\beta^* m_w/D)/(\beta q_w/k)$ for UHF/UMF
Nu_E	average Nusselt number on the heated section
Nu_x	local Nusselt number, $(\partial\theta/\partial Y)_{Y=1}$ for UWT/UWC or $(1/\theta)_{Y=1}$ for UHF/UMF
P, p	dimensionless and dimensional cross-sectional mean pressure, $P=p/(\rho u_0^2)$
Pr	Prandtl number, ν/α
q_w	wall heat flux [W m^{-2}]
Q	dimensionless volume flow rate
Sc	Schmidt number, ν/D
Sh_E	average Sherwood number on the heated section
Sh_x	local Sherwood number, $(\partial C/\partial Y)_{Y=1}$ for UWT/UWC or $(1/C)_{Y=1}$ for UHF/UMF
T	temperature [K]
u, v	velocity component in x- and y-directions, respectively [m s^{-1}]
u_0	mean velocity at the entrance [m s^{-1}]
U, V	dimensionless velocity components in x- and y-directions, respectively, $U=ub^2/(l\nu\, Gr_L)$, $V=vb/\nu$
x, y	coordinate system [m]
X, Y	dimensionless coordinate system, $X=x/(l\, Gr_L)$, $Y=y/b$
X'	fully developed flow parameter.

Greek symbols

α	thermal conductivity [m^2 s^{-1}]
β, β^*	coefficient of thermal and concentration expansion, respectively
ΔT	characteristic temperature difference, (T_w-T_0) for UWT/UWC or $q_w b/k$ for UHF/UMF
θ	dimensionless temperature, $(T-T_0)/\Delta T$
ν	kinematic viscosity [m^2 s^{-1}]
ρ	fluid density [kg m^{-3}].

Subscripts

w	value at wall
0	condition at inlet
1	unheated exit section
2	heated section
3	unheated entry section.

different type of equation. Similarity transformation is no more possible and the solution process becomes a task of great difficulty. Nelson and Wood [11–13] numerically and analytically investigated the combined heat and mass transfer for both boundary conditions. Lin and Wu [14,15] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical chemical species in dilute solutions and aqueous solutions.

The studies reviewed above deal with a uniformly heated plate (at uniform wall temperature/uniform wall concentration or uniform heat flux/uniform mass flux), while less attention is given to the study of discrete heating conditions at the surface forming the channel. The natural convection without mass transfer in a partially heated vertical channel has been studied by numerous researchers [16–20]. Results indicate that the flows with small plate-to-plate spacing are severely affected by the presence of an unheated entry. In addition, if the unheated exit is long enough, the heat transfer could be increased by 50% due to the chimney effect.

The flow geometry of interest is depicted in Fig. 1 which shows an open vertical parallel plate channel of height l and width $2b$. The fluid that enters the channel from the bottom at temperature T_0 and concentration c_0 is assumed to have uniform velocity profile u_0. The primary interest is to study the effect of both unheated entry and unheated exit on the heat and mass transfer process which is not found in the literature.

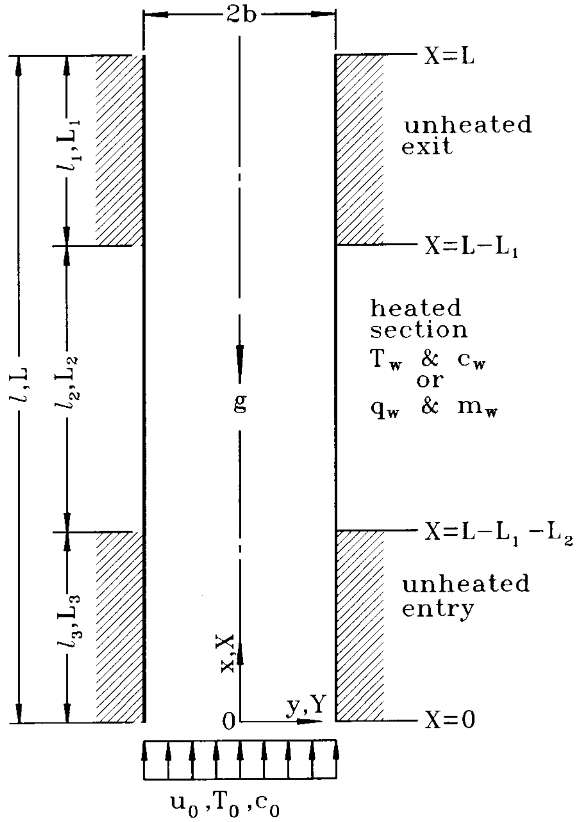

Fig. 1. Schematic diagram of the physical model.

2. Analysis

In the following analysis, it will be assumed that the flow is completely laminar and in an upward direction. The additional simplifying assumptions are made: (1) the fluid is Newtonian, (2) fluid properties, except density in buoyancy terms, are independent of temperature and concentration, (3) the Boussinesq approximation is used to characterize the buoyancy effects, (4) the flow is steady, incompressible and boundary-layer type, (5) the viscous dissipation are negligible, (6) the velocity of air entering the channel is uniform and (7) the surface normal velocity is ignored due to the small temperature and concentration difference [4,5].

Based on the above assumptions, the equations of continuity, momentum, energy and concentration for the present problem can be described in the following dimensionless form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\mathrm{d}P}{\mathrm{d}X}+\frac{\partial^2 U}{\partial Y^2}+\theta+NC \tag{2}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{3}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{4}$$

In this work, the variables of interest are the induced volume flow rate, and the average Nusselt and Sherwood numbers. If an initial velocity (i.e. the velocity at $X=0$) is termed u_0 (constant, independent of Y) and the heating of the channel is such that the change in density of the fluid can be considered to be negligible (except for the density variation in giving rise to the buoyancy force), then the dimensionless induced volumetric flow rate which is given by

$$Q=U_0=\int_0^1 U\,\mathrm{d}Y \tag{5}$$

will be constant throughout the length of the channel. Eq. (5) together with Eqs. (1)–(4) govern the natural-convective flow of the fluid throughout the channel. The average Nusselt number is the mean value of the local Nusselt number on the heated wall, i.e.,

$$Nu_{\mathrm{E}}=\frac{1}{L_2}\int_{L-L_1-L_2}^{L-L_1} Nu_x\,\mathrm{d}X \tag{6}$$

Similarly, the average Sherwood number is defined as

$$Sh_{\mathrm{E}}=\frac{1}{L_2}\int_{L-L_1-L_2}^{L-L_1} Sh_x\,\mathrm{d}X \tag{7}$$

2.1. Fully developed flow solution

If the channel wall-to-wall spacing is small relative to the channel height, the flow between the plates will become fully developed. Under these conditions, Eqs. (1)–(4) may be simplified. Assume that the flow between the plates has a parabolic velocity profile, $U=a(1-Y^2)$, where a is a constant. Substituting the equation of velocity profile in to Eq. (2) and integrating over the channel height gives

$$\int_0^L\left(-2a-\frac{\mathrm{d}P}{\mathrm{d}X}+\theta+NC\right)\mathrm{d}X=0 \tag{8}$$

2.1.1. Uniform wall temperature/uniform wall concentration (UWT/UWC)

For the UWT/UWC case, the dimensionless temperature and concentration of the air flowing between the plates will approach to unity in a relatively short distance. For $X < L-L_1-L_2$, $\theta=0$, $C = 0$ and dP/dX is constant. For $X > L-L_1-L_2$, $\theta=1$, $C = 1$ and dP/dX is again constant. The integration of Eq. (8) yields $a=(1+N)(E_1+E_2)/2$; hence, the dimensionless induced volume flow rate is given as

$$Q = \tfrac{1}{3}(1+N)(E_1+E_2) \tag{9}$$

The average Nusselt number within the heated section can be written as

$$Nu_E = \frac{Pr\,Gr_L}{E_2}\int_0^1 U\theta\,dY = \frac{(1+N)(E_1+E_2)}{3}Pr\,Gr_E$$

$$= \frac{(1+N)(1+E_1/E_2)}{3}Pr\,Gr_L \tag{10}$$

Similarly, the average Sherwood number can be obtained

$$Sh_E = \frac{Sc\,Gr_L}{E_2}\int_0^1 UC\,dY = \frac{(1+N)(E_1+E_2)}{3}Sc\,Gr_E$$

$$= \frac{(1+N)(1+E_1/E_2)}{3}Sc\,Gr_L \tag{11}$$

2.1.2. Uniform heat flux/uniform mass flux (UHF/UMF)

For the UHF/UMF case, the velocity profile is invariant in the flow direction and θ and C vary linearly with X. The bulk fluid temperature, bulk fluid concentration and pressure gradient along the channel can be expressed as follows [13]:

$$0 \le X < L - L_1 - L_2: \quad \theta_b = 0, \quad dP/dX = \text{constant}$$

$$L - L_1 - L_2 \le X < L - L_1 - L_2:$$

$$\theta_b = [X - (L - L_1 - L_2)]/(Pr\,Q),$$

$$C_b = [X - (L - L_1 - L_2)]/(Sc\,Q)$$

$$dP/dX = [\theta_b - L_2/(2\,Pr\,Q)] + N[C_b - L_2/(2\,Sc\,Q)]$$

$$L - L_1 \le X < L: \quad \theta_b = L_2/(Pr\,Q), \quad C_b = L_2/(Sc\,Q), \quad dP/dX = \text{constant} \tag{12}$$

Substituting these relations in Eq. (8), with the aid of Eq. (5), the dimensionless induced volume flow rate is written as

$$Q = \left[\frac{E_2^2(1+2E_1/E_2)}{6\,Pr\,Gr_L}\left(1+\frac{N}{Le}\right)\right]^{1/2} \tag{13}$$

Inspection of the results in Ref. [13], reveals that the dimensionless wall temperature θ_w along the heated section is

$$\theta_w = \frac{1}{Pr\,Q}[X - (L - L_1 - L_2)] + \frac{17}{35} \tag{14}$$

By substituting Eq. (14) into Eq. (6), the average Nusselt number becomes

$$Nu_E = \left[\frac{Pr\,Gr_L(1+2E_1/E_2)}{6}\left(1+\frac{N}{Le}\right)\right]^{1/2}$$

$$\ln\left\{1 + \frac{35}{17}\left[\frac{6}{Pr\,Gr_L(1+2E_1/E_2)(1+N/Le)}\right]^{1/2}\right\} \tag{15}$$

Similarly, the dimensionless wall concentration C_w along the heated section is

$$C_w = \frac{1}{Sc\,Q}[X - (L - L_1 - L_2)] + \frac{17}{35} \tag{16}$$

The average Sherwood number becomes

$$Nu_E = \left[\frac{Sc^2\,Pr\,Gr_L(1+2E_1/E_2)}{6}\left(1+\frac{N}{Le}\right)\right]^{1/2}$$

$$\ln\left\{1 + \frac{35}{17}\left[\frac{6}{Sc^2\,Pr\,Gr_L(1+2E_1/E_2)(1+N/Le)}\right]^{1/2}\right\} \tag{17}$$

3. Solution method

Because the flow under consideration is a boundary-layer-type, the system of equations was discretized to a linear system by the use of a finite difference approximation. The detailed solution procedures are similar to those of Ref. [19]. In the present study, 51 nodes in the transverse direction were used, while the grid nodes in the axial direction range from 875 to 2151, depending on the Grashof number and the length of unheated entry, heated section and unheated exit. During the program test, several grid's sizes were employed. When the grid points were doubled, the variations of the average Nusselt number Nu_E and Sherwood number Sh_E vary less than 1%. To further validate the numerical scheme used, the predicted results for a fully developed condition with various combinations of governing parameters are compared with the closed form solution. It is found that the deviations between the numerical results and analytical solutions are less than 1%. To further check the adequacy of the numerical scheme, the limiting results of natural convection heat and mass transfer over the full channel length are obtained. Excellent agreement between the present predictions and those of Nelson and Wood [11] was found. The above numerical tests indicate that the solution procedures adopted are suitable for the present study.

4. Results and discussion

In this study, there are five dimensionless groups, namely the Prandtl number Pr, the Grashof number Gr, the buoyancy ratio N, the Schmidt number Sc, and two length ratios E_1 (ratio of unheated exit length to full channel length) and E_2 (ratio of heated section to full channel length). During the course of the numerical calculations in this paper, results are obtained from the numerical solutions of air (Pr = 0.7) for both boundary conditions of UWT/UWC and UHF/UMF. The ranges of the Grashof number cover the limits of fully developed flow (small plate-spacing-to-plate-height, low Gr_L) to a single vertical plate limit (high plate-spacing-to-plate-height, high Gr_L). The Schmidt number Sc, is chosen to have the values of 0.2 (hydrogen), 0.6 (water) and 1.3 (ethyl alcohol) for the buoyancy ratio N ranging from 0 to 2.

4.1. UWT/UWC

The effects of the buoyancy ratio N and the Schmidt number Sc on the dimensionless volume rate Q with various discrete length ratios are shown in Figs. 2(a) and (b), respectively. Clearly, a greater Q is experienced for a system with a greater N as shown in Fig. 2(a). This is due to the fact that the contribution of mass fraction by the buoyancy force increases the fluid velocity. In Fig. 2(b), for a fixed value of N, the fully developed flow rate is independent of the value of Sc, but increases with an increasing value of (E_1+E_2). But in the case of developing flow, the decrease in Q with an increasing value of Sc may be contributed to the fact that increasing Sc decreases the velocity levels for buoyancy effects since the species diffusion buoyancy effect is confined more deeply in the boundary-layer region. In line with Fig. 2(a), the flow rate increases with channel length until a fully-developed value is reached. The asymptotic value of Q under fully developed condition for the UWT/UWC case depends on the value of $(1+N)(E_1+E_2)$ which is consistent with the analytical solution, Eq. (9). In addition, the Q for the case with a longer unheated entry is smaller than that for the case with a shorter unheated entry. This is expected because a longer unheated entry causes a severe flow restriction. It is also noted that a longer

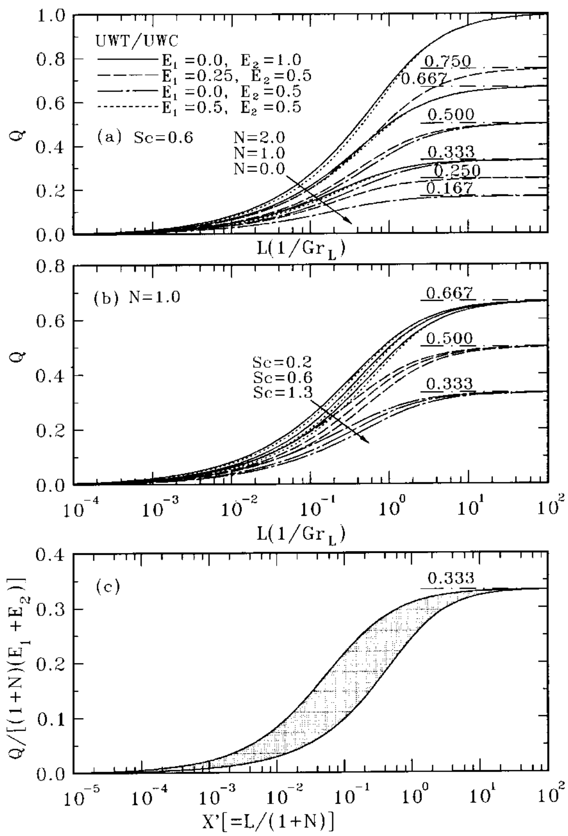

Fig. 2. Effects of buoyancy ratio N and Schmidt number Sc on dimensionless induced volume rate for the UWT/UWC case.

unheated exit induced a larger flow rate due to a greater chimney effect. To facilitate the present numerical solutions and further investigate the effects of discrete unheated and heated sections on the natural convection heat and mass transfer, the ratios of the length of unheated exit to the full channel length were assigned to be 0.0, 0.2, 0.4, 0.5, 0.6 and 0.8, while the ratios of the heated section length are chosen to have the values of 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0. The solutions of volume flow rate of various combinations of E_1, E_2, N and Sc are plotted in terms of the fully developed parameter $Q/[(1+N)(E_1+E_2)]$ vs X' $[=L/(1+N)]$. All of the results converge to the universal fully developed value of 0.333 for $X'=100$, as shown in Fig. 2(c). But each combination of the governing parameters behave differently at lower X', and the discrepancy of $Q/[(1+N)(E_1+E_2)]$ for $X' < 10^{-5}$ is nearly indistinguishable. All of the results are distributed within the shadow zone, the upper and lower limits of the shadow zone can be correlated by the following equations,

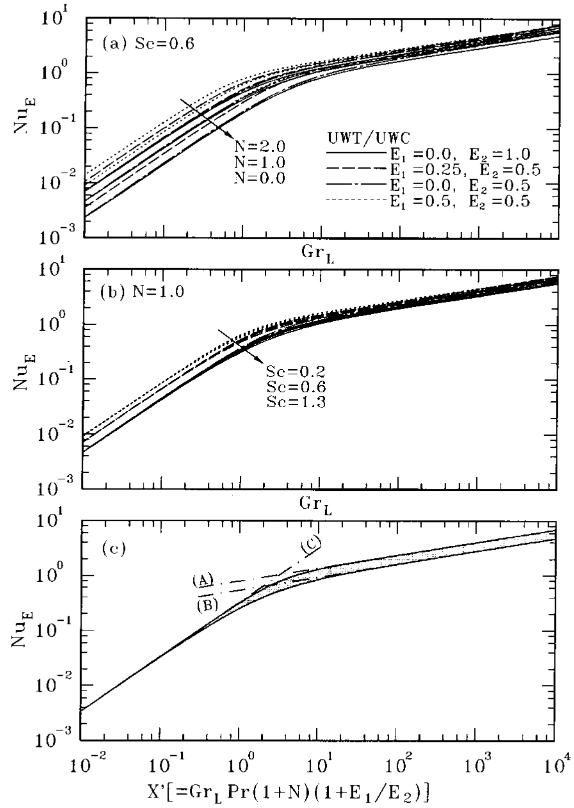

Fig. 3. Effects of buoyancy ratio N and Schmidt number Sc on the average Nusselt number for the UWT/UWC case.

$$\frac{Q}{(1+N)(E_1+E_2)} = \frac{0.333}{(1+0.111X'^{-0.868})^{1/1.4}} \tag{18}$$

for upper limit

$$\frac{Q}{(1+N)(E_1+E_2)} = \frac{0.333}{(1+0.857X'^{0.11})^{1/2}} \tag{19}$$

for lower limit

Figs. 3(a) and (b) present the variations of the average Nusselt number within the heated section Nu_E with various values of N and Sc, respectively. For $N = 0$, there is no species diffusion and the Nu_E is for natural convection from thermal buoyancy force only and is independent of the Schmidt number. When the buoyancy force form species diffusion assists the thermal buoyancy force ($N > 0$), it is noted that an increase in N results in a higher Nusselt number. In other words, the effect of increasing the buoyancy force due to mass transfer is an increase in the heat transfer rate. As shown in Fig. 3(b), for a fixed value of N, the asymptotic value of Nu_E under a fully developed condition is independent of Sc, but increases with increasing value of $(1+E_1/E_2)$. In the developing flow regime, a larger Sc results in a smaller Nu_E. This is due to the fact that the increase in Sc causes a decrease in the magnitude of velocity and hence, diffusion dominates over convection. This causes an increase in the thermal boundary layer with a corresponding smaller temperature gradient at the wall and hence, a smaller surface heat transfer rate. The important feature projected by the Nu_E–Gr_L relationship in Figs. 3(a) and (b) is that the Nu_E increases with an increasing value of $(1+N)(1+E_1/E_2)$ and the curves with the same value of $(1+N)(1+E_1/E_2)$ collapse to a single curve at low Gr_L which is consistent with Eq. (10). It is also observed that for a fixed heated section, a larger Nu_E is noted for the system with a longer unheated exit. This is simply because the channel with a longer unheated exit will draw a larger flow rate which, in turn, results in a greater heat flux in the heated section. In contrast, the system with a longer unheated entry will cause a severe restriction to the fluid flow in the channel. A careful inspection discloses that the length of unheated exit will be found to have no noticeable effect on the Nu_E at a higher Gr_L ($Gr_L > 10{,}000$) for a fixed heated section length. This is due to the fact that a larger plate-to-plate spacing chimney effect is less sensitive to the presence of an unheated exit. To facilitate the application of the results, the Nu_E is plotted against the fully developed parameter X' [$X' = Pr\ Gr_L(1+N)(1+E_1/E_2)$], as shown in Fig. 3(c). It is of interest to note that for a large value of G_L, the slope of the curve approaches the 1/4 power law of the heated vertical plate limit. The dotted lines (A) and (B) on the graph can be represented by the equations of $Nu_E = 0.804X'^{0.233}$ and $Nu_E = 0.554X'^{0.233}$, respectively. A curve fit for Nu_E for low X' is well represented by the dotted line (C) with $Nu_E = 0.33X'$. The limiting correlation equations can be written as

$$Nu_E = \frac{0.33X'}{(1+0.118X'^{1.829})^{1/2.4}} \quad \text{for upper limit} \tag{20}$$

$$Nu_E = \frac{0.33X'}{(1+0.484X'^{1.067})^{1/1.4}} \quad \text{for lower limit} \tag{21}$$

The effects of N and Sc on the Sherwood number are shown in Figs. 4(a) and (b), respectively. In Fig. 4(a), the Sh_E increases when the buoyancy force from mass diffusion assists the thermal buoyancy flow. This is owing to the fact that an increase in the N will result in a thinning of the concentration boundary layer. Similar to the results of Nu_E, for a fixed value of Sc the fully-developed value Sh_E for various values of $(1+E_1/E_2)(1+N)$ collapses to a single asymptote and the discrepancy of Sh_E at $Gr_L > 10{,}000$ for a fixed E_2 is nearly indistinguishable. The effect of Sc on the

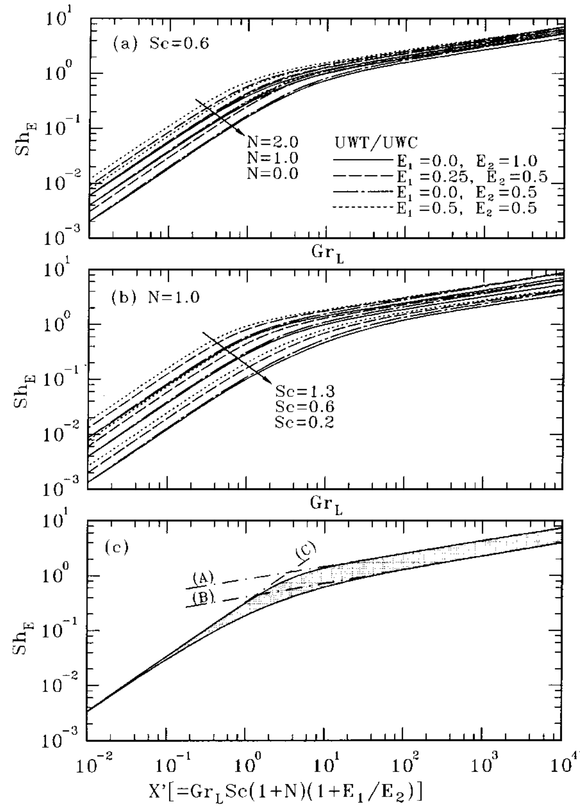

Fig. 4. Effects of buoyancy ratio N and Schmidt number Sc on the average Sherwood number for the UWT/UWC case.

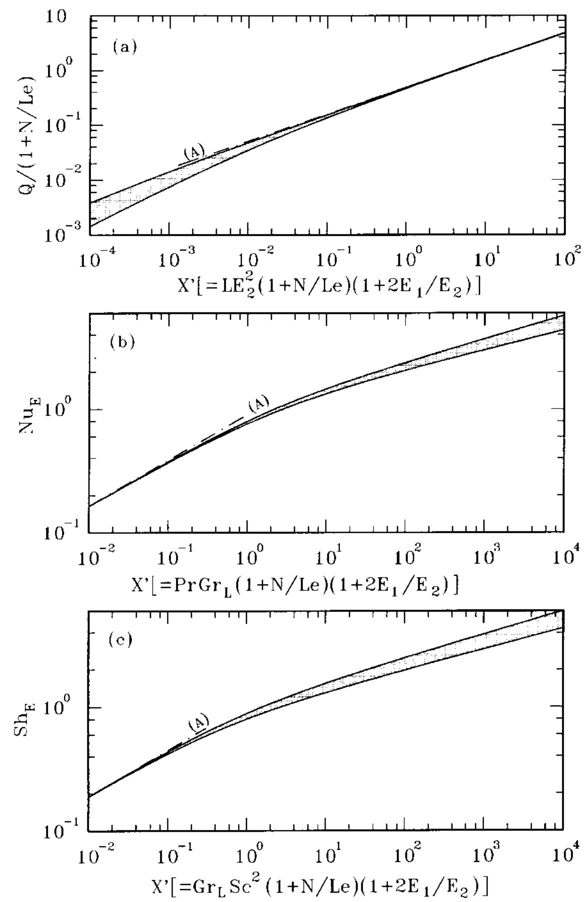

Fig. 5. The upper and lower limits of a dimensionless induced volume rate, and the average Nusselt and Sherwood numbers for the UHF/UMF case.

Sherwood number is depicted in Fig. 4(b). In contrast to the trend of Nu_E, larger values of Sc are seen to provide larger Sh_E. This can be explained from the fact that a larger Sc corresponds to a smaller diffusion coefficient for a given fluid and thinner concentration boundary layer relative to the flow boundary layer. This results in a larger concentration gradient at the wall and hence, a larger mass diffusion or larger Sherwood number. It can be concluded from Figs. 4(a) and (b) that the Sh_E increases with an increasing value of $Sc(1+N)(1+E_1/E_2)$. Due to the same reason discussed in the Nu_E results, the Sh_E is larger for a longer unheated exit, but smaller for a longer unheated entry. The relationship of Sh_E with fully a developed parameter X' $[X' = Gr_L\ Sc(1+N)(1+E_1/E_2)]$ is shown in Fig. 4(c). The curve fits for upper and lower limits of Sh_E in the X' range of 100 to 10,000 are well predicted by $0.864X'^{0.233}$ [dotted line (A)] and $0.46X'^{0.233}$ [dotted line (B)]. An excellent curve fit for Sh_E under the fully developed condition is obtained with the equation of $0.3X'$ [dotted line (C)]. The upper and lower limits of Sh_E in Fig. 4(c) can be correlated by

$$Sh_E = \frac{0.3X'}{(1+1.043X'^{1.791})^{1/2.35}} \quad \text{for upper limit} \tag{22}$$

$$Sh_E = \frac{0.3X'}{(1+0.730X'^{0.724})^{1/0.95}} \quad \text{for lower limit} \tag{23}$$

4.2. UHF/UMF

Consideration is now given to the UHF/UMF case. Similar to the UWT/UWC case, Figs. 5(a)–(c) show the variations of Q, Nu_E and Sh_E with the fully-developed flow parameters. In Fig. 5(a), the results $Q/(1+N/Le)$ against X' $[X' = LE_2^2(1+N/Le)(1+2E_1/E_2)]$ nearly converge to a universal curve which can be expressed by the equation $Q = 0.49X'^{0.5}$ [dotted line (A)]. A limiting equation to correlate the relationship between Q and X' for various combinations of governing parameters can be expressed as

$$\frac{Q}{(1+N/Le)} = \frac{0.49X'^{0.5}}{(1+0.00697X'^{-0.6})^{1/4}} \tag{24}$$

for upper limit

$$\frac{Q}{(1+N/Le)} = \frac{0.49X'^{0.5}}{(1+0.169X'^{-0.462})^{1/2.2}} \tag{25}$$

for lower limit

The relationship of Nu_E vs X' [$X' = Pr\ Gr_L(1+N/Le)(1+2E_1/E_2)$] is shown in Fig. 5(b). In this case, the results of the smaller X' collapse to a single asymptotic curve. The correlating equation for the fully developed regime can be given by $Nu_E = 0.9X'^{0.37}$ [dotted line (A)]. In Fig. 5(b), the limiting correlation equations can be written as

$$Nu_E = \frac{0.9X'^{0.37}}{(1+0.624X'^{0.756})^{1/4.2}} \quad \text{for upper limit} \tag{26}$$

$$Nu_E = \frac{0.9X'^{0.37}}{(1+0.714X'^{0.672})^{1/3.2}} \quad \text{for lower limit} \tag{27}$$

In Fig. 5(c), the result of Sh_E under a fully developed condition is well correlated by $Sh_E = 1.1X'^{0.38}$ [$X' = Gr_L\ Sc^2(1+N)(1+2E_1/E_2)$]. The limiting correlation equation for various combinations of governing parameters in Fig. 5(c) may be written as

$$Sh_E = \frac{1.1X'^{0.38}}{(1+1.189X'^{0.7031})^{1/3.7}} \quad \text{for upper limit} \tag{28}$$

$$Sh_E = \frac{1.1X'^{0.38}}{(1+1.87X'^{0.693})^{1/3.3}} \quad \text{for lower limit} \tag{29}$$

In addition, a comparison of separate numerical results found that Q, Nu_E and Sh_E increase with an increasing value of $E_2^2(1+N/Le)(1+2E_2/E_1)$, $(1+N/Le)(1+2E_1/E_2)$ and $Sc^2(1+N/Le)(1+2E_1/E_2)$, respectively.

5. Conclusions

The natural convection heat and mass transfer in vertical parallel plates with discrete heating has been studied. Analytical solutions of the dimensionless volume flow rate, the average Nusselt number and the average Sherwood number are also derived under fully developed flow conditions for both UWT/UWC and UHF/UMF boundary conditions.

What follows is a brief summary of major results:

1. In the case of UWT/UWC, Q, Nu_E and Sh_E increase with an increasing value of $(1+N)(E_1+E_2)$, $(1+N)(1+E_1/E_2)$ and $Sc(1+N)(1+E_1/E_2)$, respectively.
2. In the case of UHF/UMF, Q, Nu_E and Sh_E increase with an increasing value of $E_2^2(1+N/Le)(1+2E_1/E_2)$, $(1+N/Le)(1+2E_1/E_2)$ and $Sc^2(1+N/Le)(1+2E_1/E_2)$, respectively.
3. Under fully developed conditions, Q, Nu_E and Sh_E can be derived analytically and be expressed as $(1+N)(E_1+E_2)/3$, $Pr\ Gr_L(1+N)(1+E_1/E_2)/3$ and $Sc\ Gr_L(1+N)(1+E_1/E_2)/3$ for the UWT/UWC case, respectively.

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